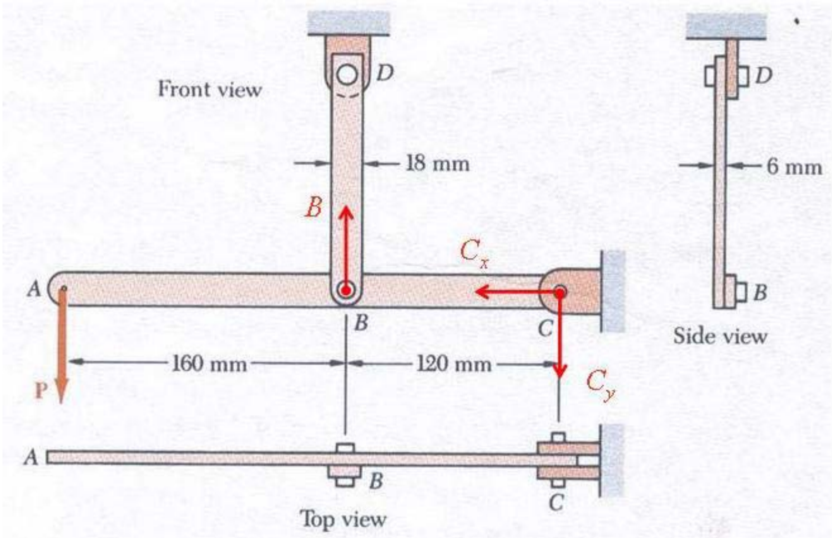
Simple Stress
(Strength of Materials - I, Midterm Exam-40-1)

Problem :



1. A steel structure

In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3 is desired, determine the largest load P which may be applied at A. Note that link BD is not reinforced around the pin holes.

Solution :

The allowable shear stress

$$\tau_{al} = \frac{\tau_{ul}}{FS} = \frac{150}{3} = 50 \text{ MPa} \quad (1)$$

The allowable normal stress

$$\sigma_{al} = \frac{\sigma_{ul}}{FS} = \frac{400}{3} = 133.3 \text{ MPa} \quad (2)$$

Equilibrium equation

$$\begin{aligned} \sum M_C &= 0, \quad -120 \times F_{BD} + 280 \times P = 0 \\ P &= \frac{3}{7} F_{BD} \end{aligned} \quad (3)$$

i) BD link

$$\begin{aligned} \sigma_{al} &= \left(\frac{F}{A}\right)_{BD}, \quad F_{BD} = \sigma_{al} \times A_{BD} \\ F_{BD} &= 133.3 \times (18 - 10) \times 6 \\ F_{BD} &= 6398.4 \text{ N} \end{aligned} \quad (4)$$

$$P = \frac{3}{7} \times 6398.4, \quad P = 2742.2 \text{ N} \quad (5)$$

ii) Pin B

$$
\begin{aligned}
\tau_{al} &= \frac{F_{BD}}{A_B}, \quad F_{BD} = \tau_{al} \times A_B \\
F_{BD} &= 50 \times \left(\frac{\pi}{4} \times 10^2\right) \\
F_{BD} &= 3927.0 \text{ N} \qquad (6)
\end{aligned}
$$

$$
P = \frac{3}{7} \times 3927.0, \quad P = 1683.0 \text{ N} \qquad (7)
$$

iii) Pin C

$$
\begin{aligned}
\tau_{al} &= \frac{C}{2A_C}, \quad C = \tau_{al} \times 2A_C \\
C &= 50 \times 2\left(\frac{\pi}{4} \times 6^2\right) \\
C &= 2827.4 \text{ N} \qquad (8)
\end{aligned}
$$

Equilibrium equation

$$
\begin{aligned}
\sum F_x &= 0, \quad C_x = 0 \\
C_y &= C = 2827.4 \text{ N} \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
\sum M_B &= 0, \quad -120 \times C_y + 160 \times P = 0 \\
P &= \frac{3}{4} C_y \qquad (10)
\end{aligned}
$$

$$
P = \frac{3}{4} \times 2827.4, \quad P = 2120.6 \text{ N} \qquad (11)
$$

The allowable load P

$$
P = 1683.0 \text{ N}
$$